

# Lesson 3-4: The Polygon Angle-Sum Theorems

## Triangle Angle-Sum Theorem

Yesterday we learned that the sum of the angle measures of a triangle is 180. What about for a square or a pentagon? Today we are going to consider more complex shapes than triangles: polygons.

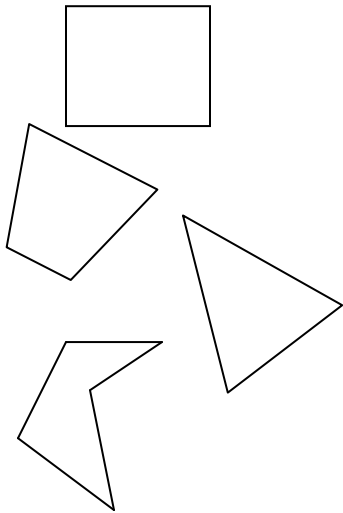
## Polygons

A polygon is a closed geometric figure in a plane formed by connecting line segments endpoint to endpoint, each segment intersecting exactly two others.

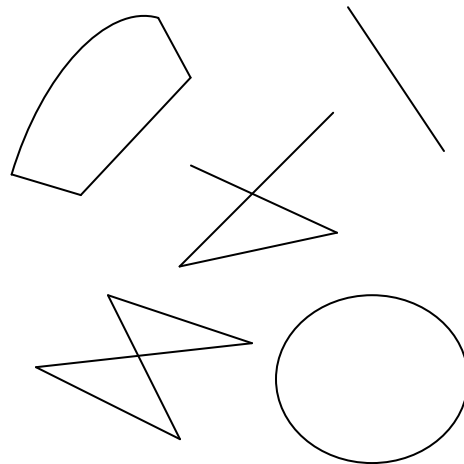
OK, that is a mouthful. Let's make it simple. A square is a polygon. It has no gaps (it is closed), its sides are segments that connect at the corners (endpoint to endpoint, exactly intersecting two others). A rectangle is another example.

Is a circle a polygon? No...it is made of a curve, not a line segment. Here are some more examples:

**Yes**

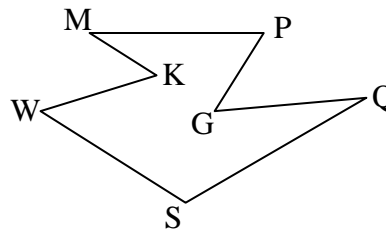
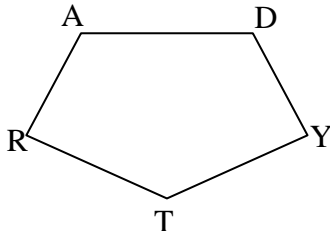


**No**



## Naming Polygons

To name a polygon, pick any vertex and list the label of each vertex in order. It doesn't matter if you go clockwise or counter-clockwise; just pick a direction and go. The name of the polygon on the left is *ADYTR* and on the right is *MKWSQGP*.



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## Classifying Polygons

We classify polygons based on two attributes:

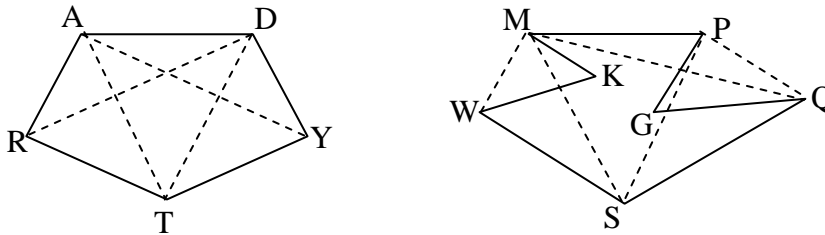
1. The number of sides.
2. Whether they are concave or convex.

### Concave or convex

To determine if a polygon is concave or convex, look at its diagonals. What is a diagonal you ask? A diagonal is a segment that connects two nonconsecutive vertices.

If all of the diagonals are completely inside the polygon, it is a convex polygon.

If any part of any diagonal is outside the polygon, it is a concave polygon. Think of it this way. A cave is a hole in a mountain side. A concave polygon has a cave in its side. Using our polygons from earlier, classify each as either concave or convex:



Polygon  $ADYTR$  is convex because all of the diagonal (all are shown) lie completely inside the polygon. Polygon  $MPGQSWK$  is concave because numerous diagonals (not all are shown) pass outside the polygon.

In this course, you may assume that a polygon is convex unless told otherwise.

### Number of sides

The main way of classifying polygons is by the number of sides. For instance you know instantly what a triangle is (a three sided polygon) and what a pentagon is (a five sided polygon). We have common names for most polygons with 12 or fewer sides. In general, you may refer to a polygon that has  $n$  sides as an  $n$ -gon. Here is a table with most of the common names:

Sides	Name
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
8	octagon
9	nonagon
10	decagon
12	dodecagon
$n$	$n$ -gon

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### Practice classifying polygons

Look at Check Understanding #2 on page 144. Classify each polygon by its sides. Identify it as concave or convex.

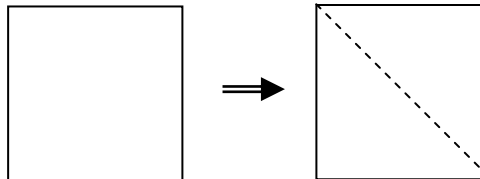
- a) hexagon; convex
- b) octagon; concave
- c) 24-gon; concave

### Polygon angle-sums

You know that the angle-sum of a triangle is 180. With very little effort you can see that the same doesn't hold true for a square. Just what is the angle-sum for a square?

Do you remember Postulate 1-10? It says the area of a region is the sum of the areas of its non-overlapping regions. Now, it doesn't directly apply here but does the basic idea apply? We know how find the angle sum of a triangle. Can we divide the square into a set of triangles? Yes we can! The square can be divided into two triangles:

This tells us the angle sum of a square is  $2 * 180$  or 360.



Repeat this with 5, 6, 7, & 8 sided polygons.

Build a table listing the number of sides, number of triangles formed, and the polygon's angle-sum. Can you make a conjecture about a polygon's angle sum?

# sides	# triangles	Sum of int. $\angle$ measures
4	2	$2 * 180 = 360$
5	3	$3 * 180 = 540$
6	4	$4 * 180 = 720$
7	5	$5 * 180 = 900$
8	6	$6 * 180 = 1080$

Looking at the pattern developing, it looks like the angle-sum of an  $n$ -gon would be:

$$(n - 2) * 180$$

### Theorem 3-9 Polygon Angle-Sum Theorem

The sum of the measures of the interior angles of an  $n$ -gon is  $(n - 2)180$ .

### Example – Pg 145, Check Understanding #3

- a) What is the sum of the measures of the angles of a 13-gon?
- b) If the angle-sum of a polygon is 720, how many sides does it have?

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- a) A 13-gon has 13 sides:  $n = 13$ , so  $(n - 2)180 = (13 - 2)180 = 11 \cdot 180 = 1980$   
b)  $(n - 2)180 = 720$ ,  $n - 2 = 4$ ,  $n = 6$  so 6 sides (a hexagon)

### Example – Pg 147, #18

The polygon has seven sides, so its angle-sum is  $(7 - 2)180 = 5 \cdot 180 = 900$ .

We are given angle measures of  $y^\circ$ ,  $125^\circ$ ,  $135^\circ$ ,  $130^\circ$ ,  $129^\circ$ ,  $116^\circ$ , and  $120^\circ$ .

Therefore,  $y + 125 + 135 + 130 + 129 + 116 + 120 = y + 755$ .

and  $y + 755 = 900$ , or  $y = 145$ .

### Polygon exterior angle-sum

If you construct and measure the external angle, one at each vertex of a polygon, you will quickly form the conjecture that the sum of external angle measures for any polygon is 360. Examine the diagrams on page 146 to visualize this. This conjecture is the next theorem.

### Theorem 3-10 Polygon External Angle-Sum Theorem

The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360:

$$m\angle 1 + m\angle 2 + m\angle 3 + \dots + m\angle n = 360$$

### Special polygon classifications

There are three special polygon types:

- Equilateral polygon: all sides congruent.
- Equiangular polygon: all angles congruent.
- Regular polygon: both equilateral and equiangular.

### Example – Pg 147, #24

Find the measures of an interior angle and an exterior angle of a regular 18-gon.

- Angle-sum is  $(18 - 2)180 = 16 \cdot 180 = 2880$
- It is a regular polygon so all angles have equal measure.
- Interior angle: There are 18 angles so  $18x = 2880$  or  $x = 160$
- Exterior angle: 18 exterior angles so  $18x = 360$ , or  $x = 20$

### Assign homework

- p. 147 #1-25 odd, 32-35, 37-44, 47-53, 59, 64-70  
p. 151 #1-9